12 – Aeroservoelasticity

Vibraciones y Aeroelasticidad

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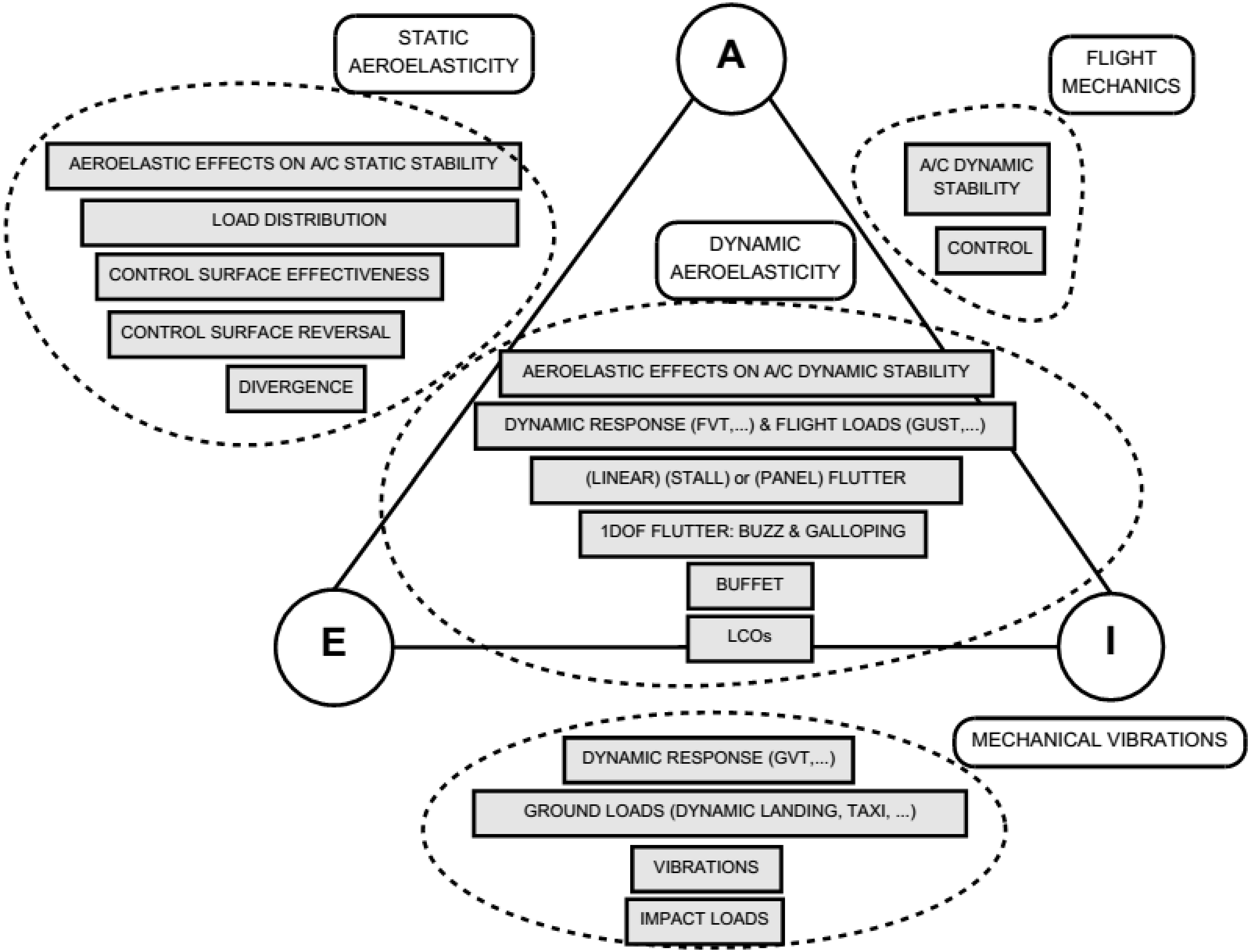
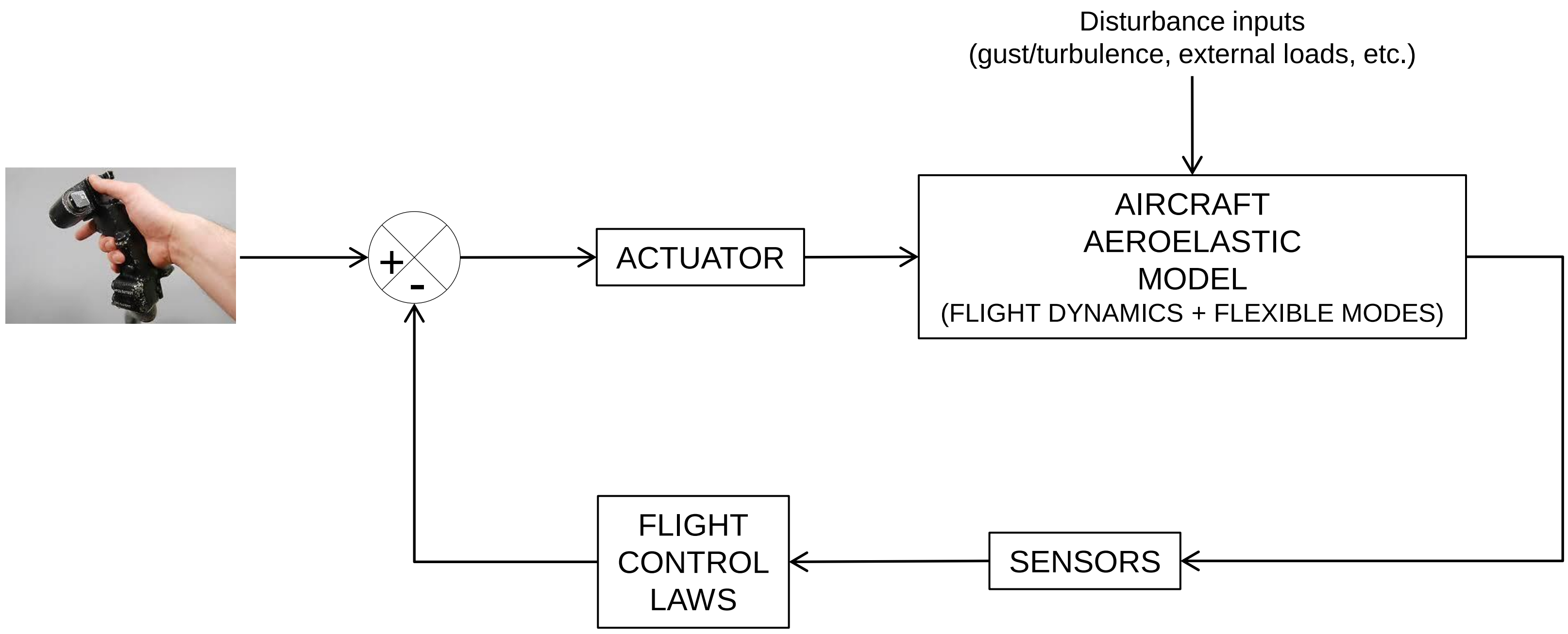
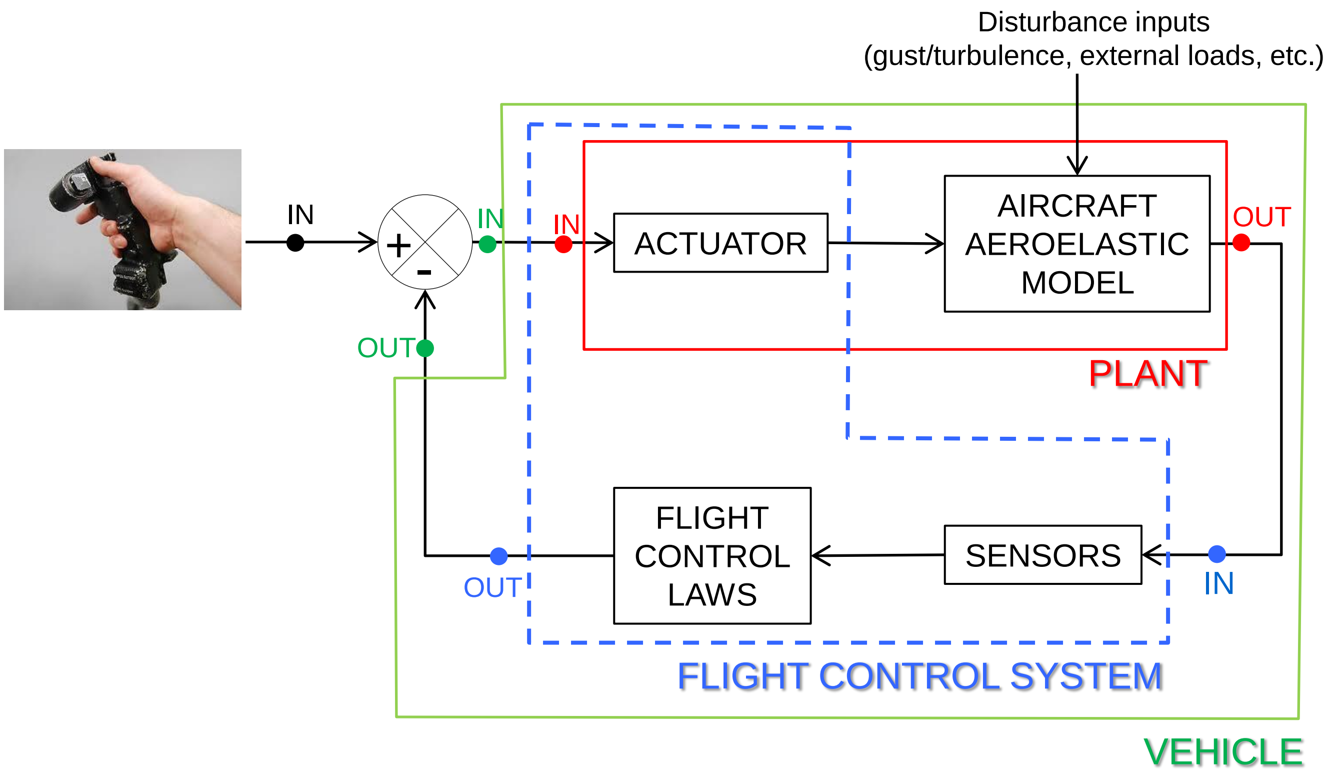
BUFFET
WHERE WE ARE IN THE COLLAR'S DIAGRAM ?



BLOCK DIAGRAM OF A TYPICAL AEROSERVOELASTIC SYSTEM



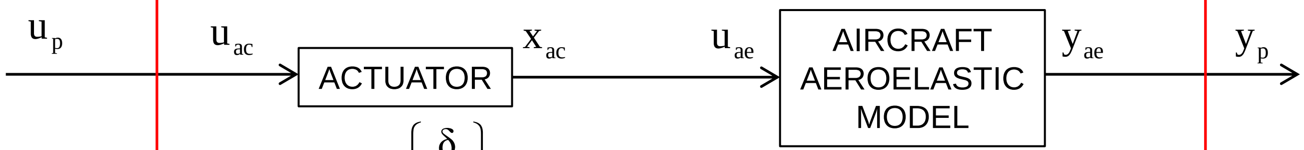
CONCEPT OF "PLANT", "FLIGHT CONTROL SYSTEM", AND "VEHICLE"



- •IN = demanded control surfaces (CS) rotations (pilot inputs)
- •IN •IN = commanded CS rotations
- •OUT •IN = sensors' inputs (displacements, velocities, and accelerations at structural points)
- •OUT •OUT = flight control laws (FCLs) outputs

PLANT EQUATIONS IN STATE-SPACE FORMAT

PLANT



$$x_{ac} = \begin{Bmatrix} \delta \\ s\delta \\ s^2\delta \end{Bmatrix} \qquad x_{ae} = \begin{Bmatrix} \xi \\ s\xi \end{Bmatrix}$$

$$\frac{\delta}{u_{ac}} = \frac{a_0}{s^3 + a_2 s^2 + a_1 s + a_0}$$

$$\frac{dx_{ac}}{dt} = A_{ac} x_{ac} + B_{ac} u_{ac}$$

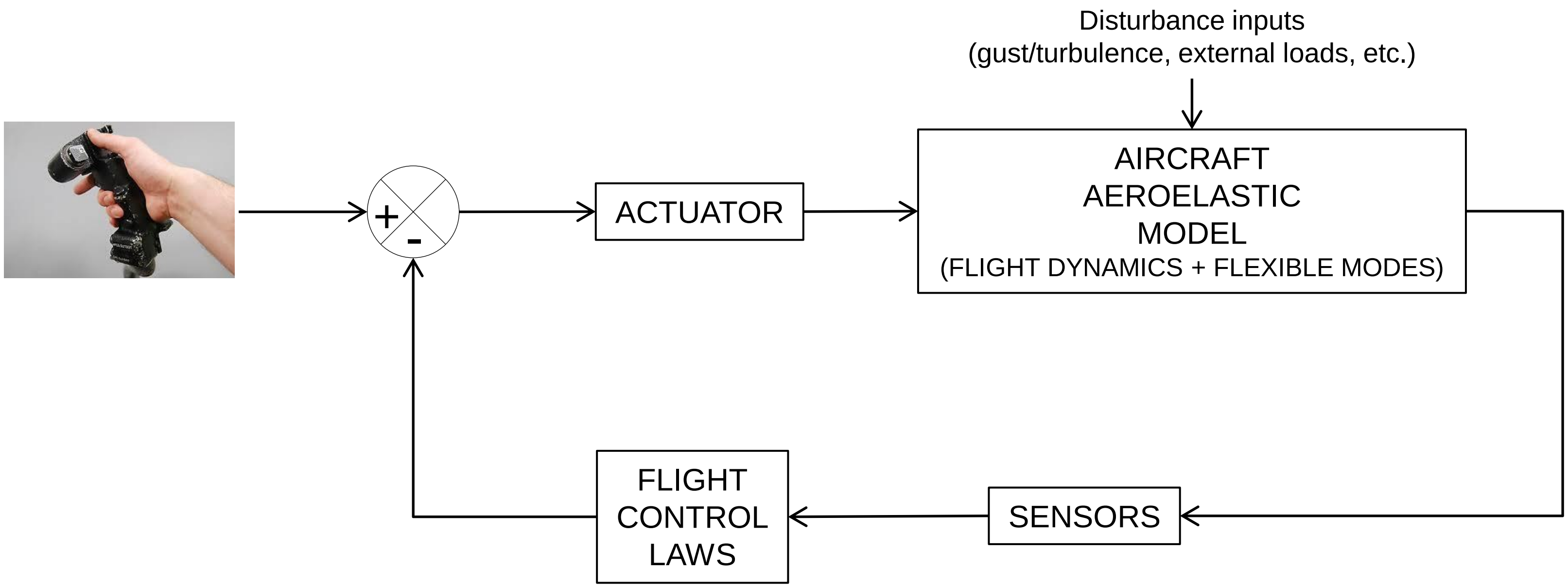
$$\frac{dx_{ae}}{dt} = A_{ae} x_{ae} + B_{ae} x_{ac}$$

$$y_{ae} = C_{ae} x_{ae} + D_{ae} x_{ac}$$

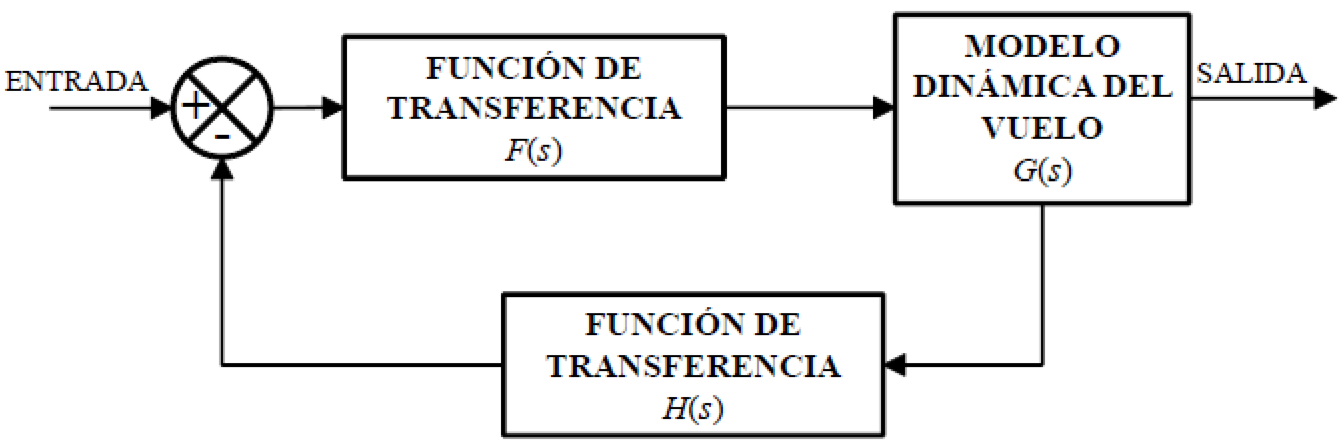
$$\frac{dx_p}{dt} = \frac{d}{dt}\begin{Bmatrix} x_{ae} \\ x_{ac} \end{Bmatrix} = \begin{bmatrix} A_{ae} & B_{ae} \\ 0 & A_{ac} \end{bmatrix}\begin{Bmatrix} x_{ae} \\ x_{ac} \end{Bmatrix} + \begin{bmatrix} 0 \\ B_{ac} \end{bmatrix} u_{ac} = A_p x_p + B_p u_p$$

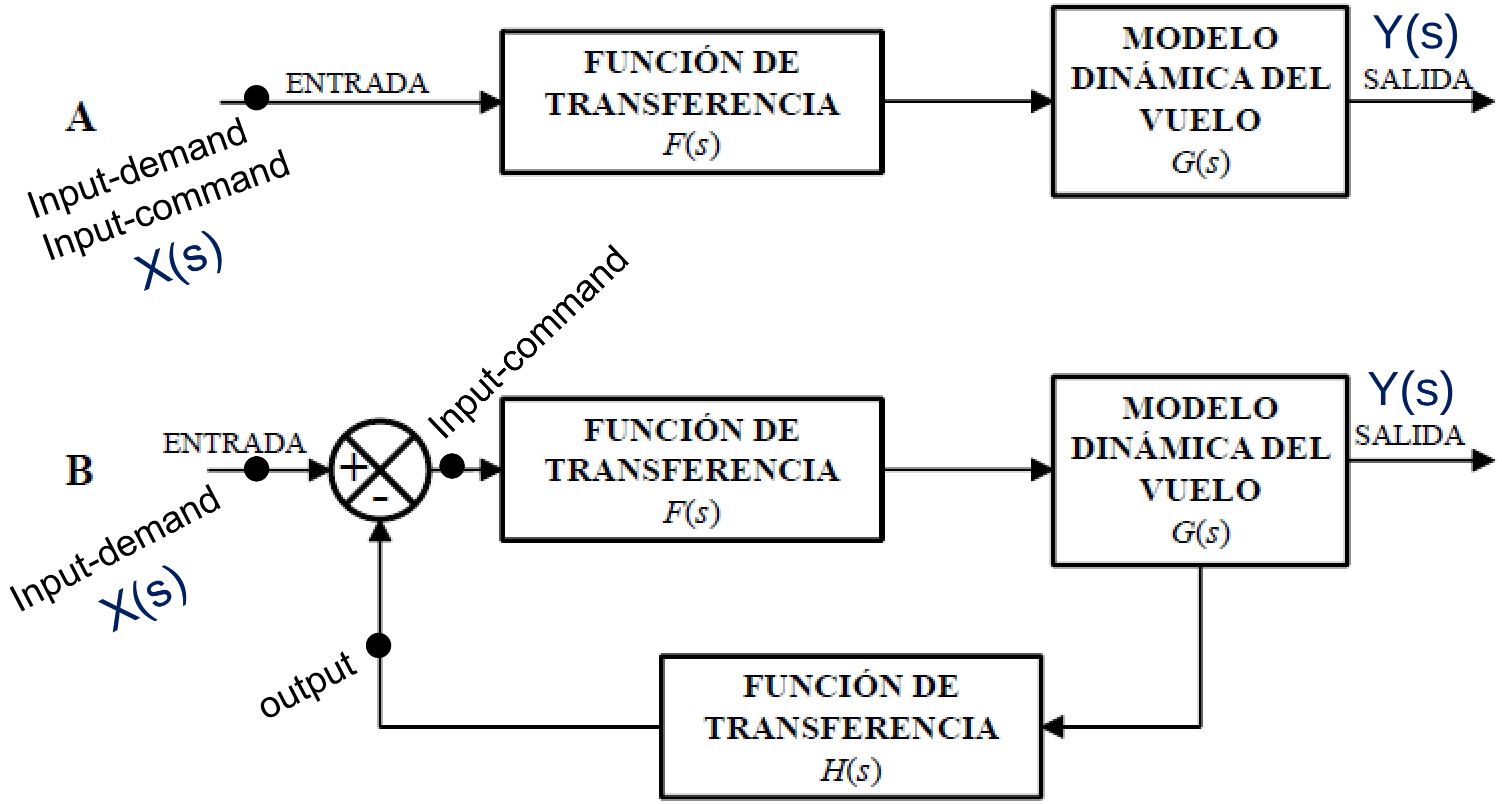
$$y_p = y_{ae} = \begin{bmatrix} C_{ae} & D_{ae} \end{bmatrix}\begin{Bmatrix} x_{ae} \\ x_{ac} \end{Bmatrix} = C_p x_p$$

SIMPLIFIED BLOCK DIAGRAM



Previous block diagram is simplified by the following scheme:



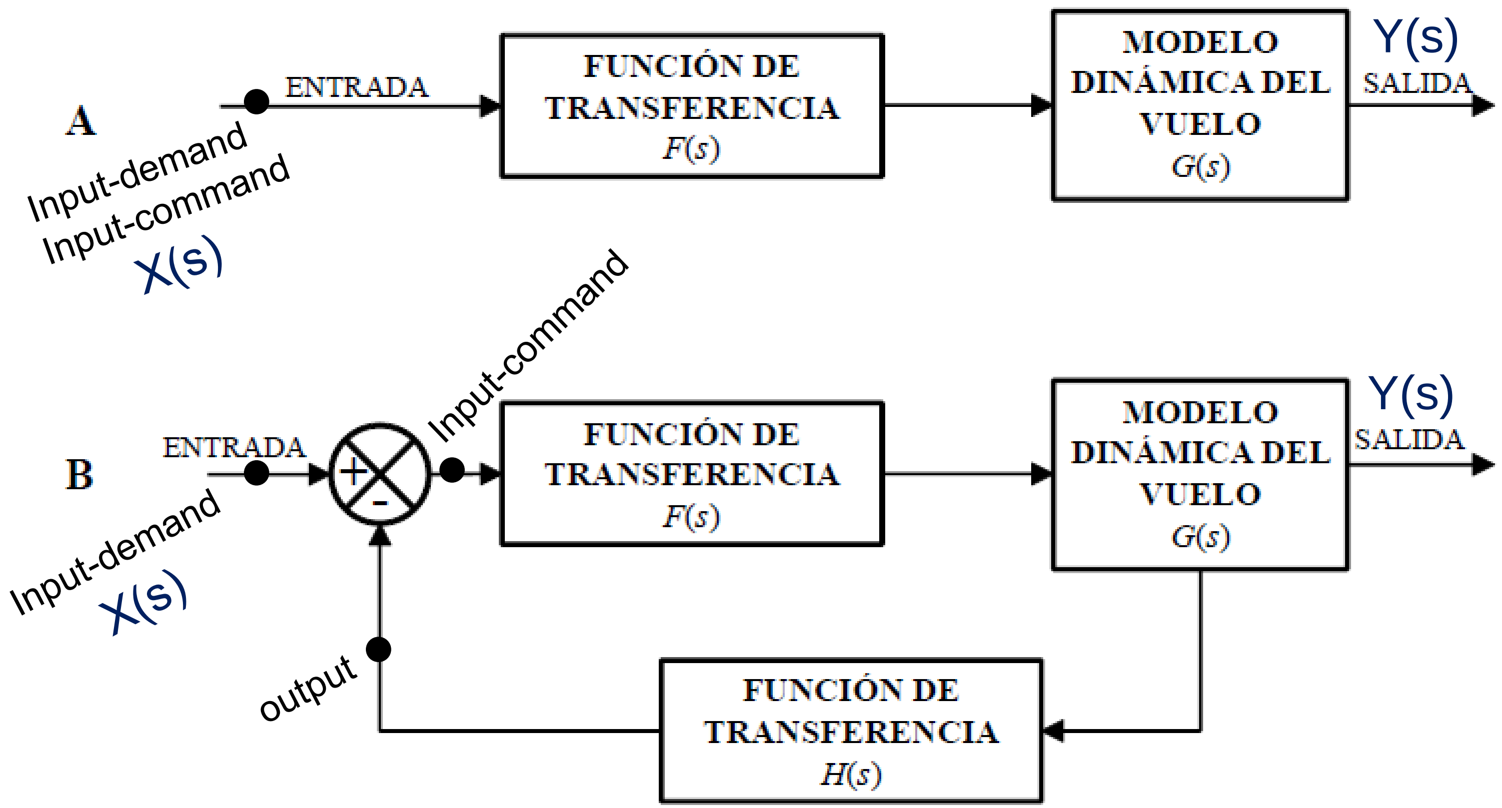


Open loop system:

- The input-demand is used as input-command, which in turn does not depend of the output

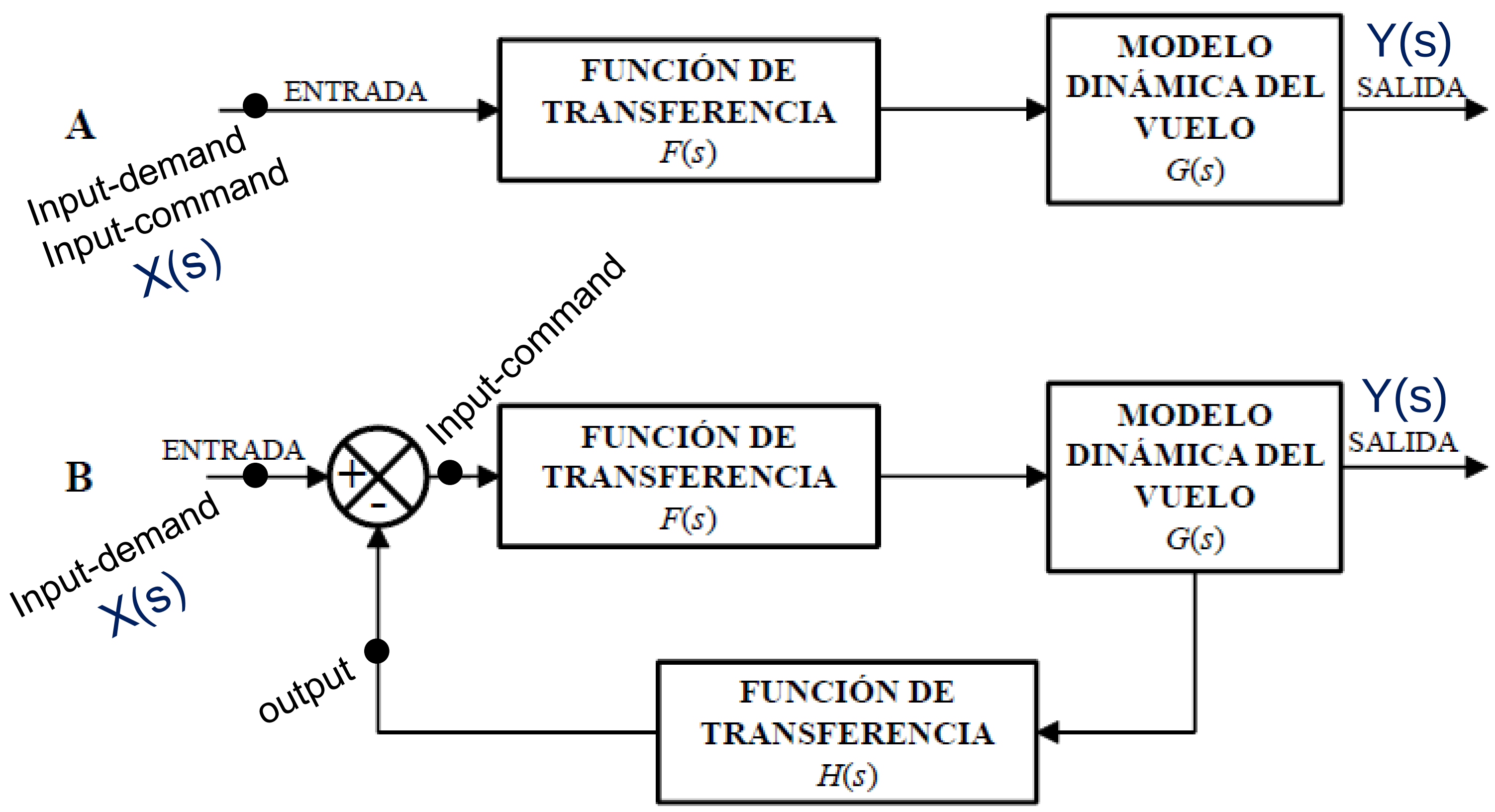
Close loop system:

- The input-demand is compared with the output (response) and its difference is used as input-command to the system.



$$TF_{\text{open}}(s) = \frac{Y(s)}{X(s)} = G(s)\,F(s)$$

$$TF_{\text{closed}} = \frac{F(s)\,G(s)}{1 - F(s)\,G(s)\,H(s)}$$



$$TF_{\text{open}}(s) = \frac{Y(s)}{X(s)} = G(s)\,F(s)$$

$$TF_{\text{closed}} = \frac{F(s)\,G(s)}{1 - F(s)\,G(s)\,H(s)}$$

Stability criteria: root-locus method of Evans

$$TF_{\text{closed}}(s) = K\frac{s^m + b_{m-1}s^{m-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} = K\frac{(s-z_1)(s-z_2)\cdots(s-z_m)}{(s-p_1)(s-p_2)\cdots(s-p_n)}$$

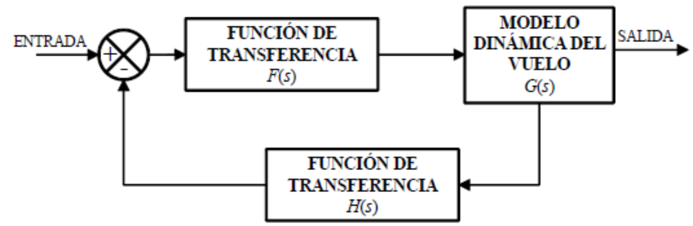
- $s=p_i$ are the “poles” and s=zi are the “zeros” of the transfer function
- The ”complex conjugate root” theorem states that if P(s) is a polynomial in one variable with real coefficients, and a + bi is a root of P(s) with a and b real numbers, then its complex conjugate a − bi is also a root of P(s). Then, the “poles” are real or complex values that are written as:

$$\sigma \pm i\omega = -\zeta\omega \pm i\omega\sqrt{1-\zeta^2} = -\zeta\omega \pm i\omega_d \approx -\zeta\omega \pm i\omega$$

- The time-domain response is therefore written as (1):

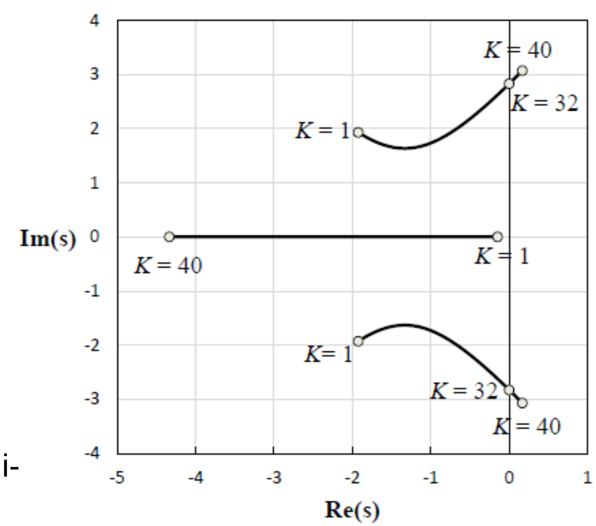
$$A \cdot e^{-\zeta\omega t} \cdot \sin(\omega t + \varphi)$$

- The system is unstable if the real part of one of the poles is positive. The graph of all possible poles with respect to some particular variable (whether system gain or some other parameter) is called the “root locus”, and the design technique based on this graph is called the root-locus method of Evans (1948) (2)



$$F(s) = 1,\ G(s) = \frac{K}{s(s^2+4s+8)} \text{ y } H(s) = 1$$

$$TF_{closed} = \frac{\dfrac{K}{s(s^2+4s+8)}}{1 + \dfrac{K}{s(s^2+4s+8)}} = \frac{K}{s^3+4s^2+8s+K}$$



(1) Sistemas realimentados de control, D’’azzo-Houpis, Ed. Paraninfo, 1992.

(2) Feedback control of dynamic systems, G.F. Franklin, J.D. Powell, and A. Emami-Naeini, Addison-Wesley Publishing Company, 1994

Stability criteria: the Nyquist plot

$$TF_{\text{closed}}(\mathrm{i}\omega) = \frac{F(\mathrm{i}\omega)\,G(\mathrm{i}\omega)}{1 + F(\mathrm{i}\omega)\,G(\mathrm{i}\omega)\,H(\mathrm{i}\omega)}$$

- For most systems, a simple relationship exists between closed-loop stability and the open-loop frequency response, i.e.,

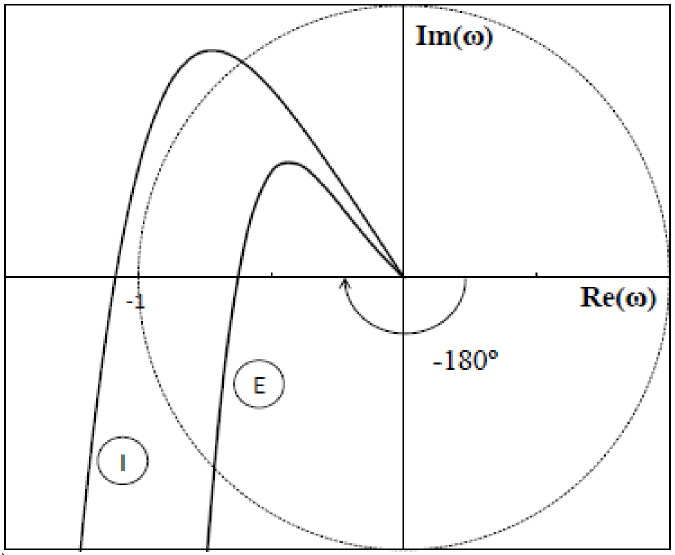
$$F(i\omega)\cdot G(i\omega)\cdot H(i\omega)$$

- All points at the intersection of the root locus with the imaginary axis (neutral stability) have the property

$$\left|F(i\omega)\cdot G(i\omega)\cdot H(i\omega)\right| = 1 \qquad \angle F(i\omega)\cdot G(i\omega)\cdot H(i\omega) = 180^{o}$$

- The stability condition based on the open-loop frequency response is:

$$\left|F(i\omega)\cdot G(i\omega)\cdot H(i\omega)\right| < 1 \quad \text{at } \angle F(i\omega)\cdot G(i\omega)\cdot H(i\omega) = -180^{o}$$



- Gain margin

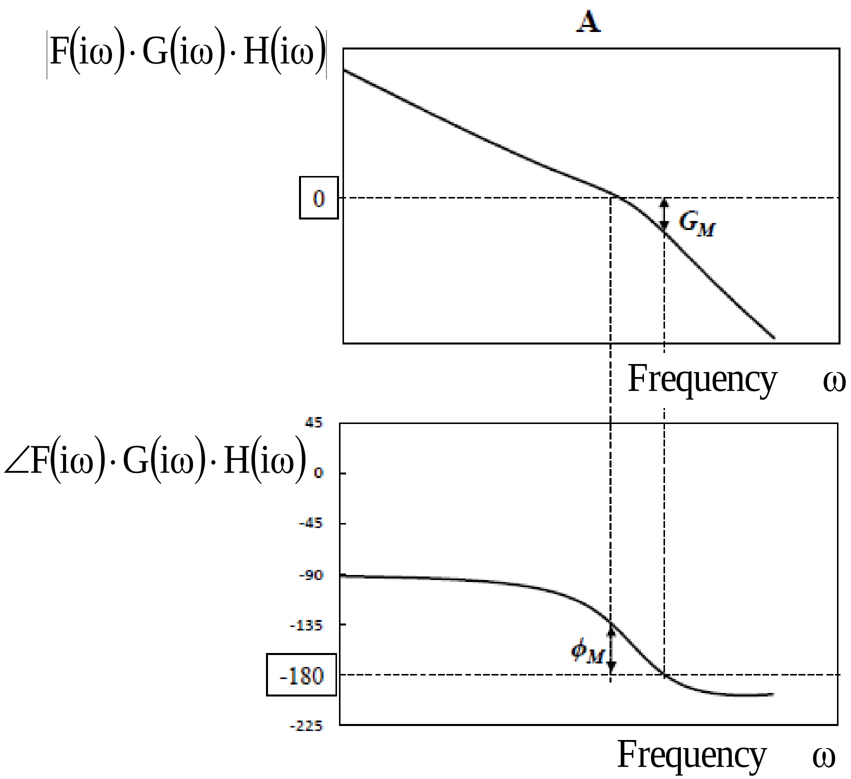
$$20\cdot\log_{10}\left|G_M\cdot F(i\omega)\cdot G(i\omega)\cdot H(i\omega)\right| = 20\log_{10}G_M + 20\log_{10}\left|F(i\omega)\cdot G(i\omega)\cdot H(i\omega)\right| = 1$$

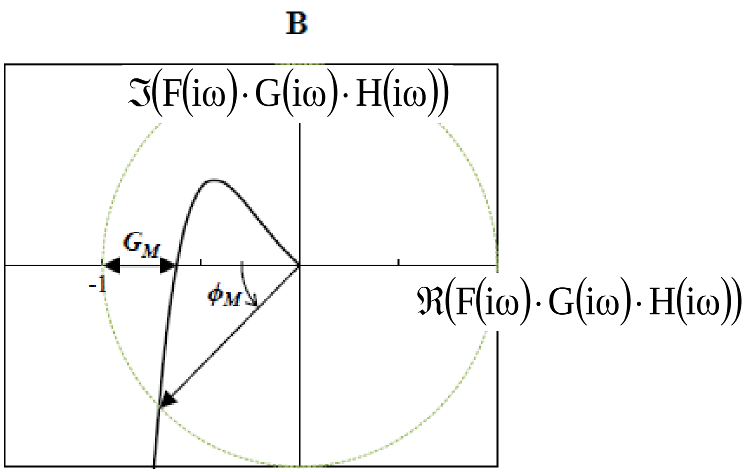
- The system is unstable if the real part of one of the poles is positive. The graph of all possible poles with respect to some particular variable (whether system gain or some other parameter) is called the “root locus”, and the design technique based on this graph is called the root-locus method of Evans (1948) (2)

Gain margin and phase margin in Bode and Nyquist plots

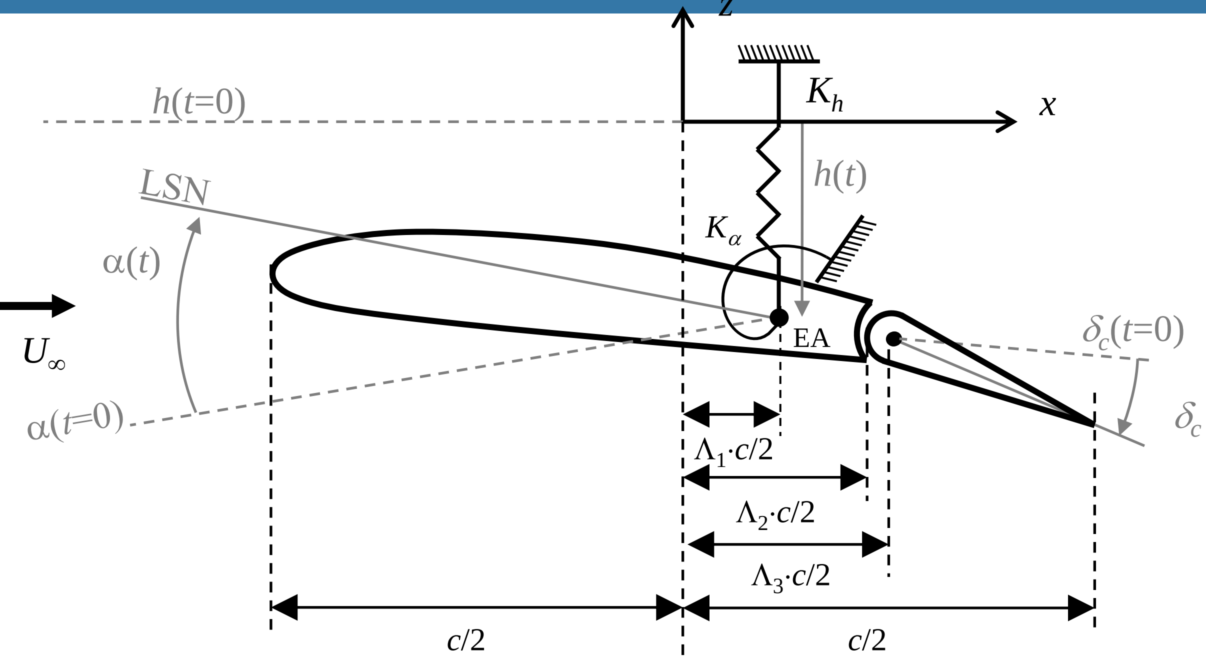
$$TF_{\text{closed}}(\mathrm{i}\omega) = \frac{F(\mathrm{i}\omega)\,G(\mathrm{i}\omega)}{1 + F(\mathrm{i}\omega)\,G(\mathrm{i}\omega)\,H(\mathrm{i}\omega)}$$

- Gain margin $20 \cdot \log_{10}\left|G_M \cdot F(i\omega) \cdot G(i\omega) \cdot H(i\omega)\right| = 20\log_{10} G_M + 20\log_{10}\left|F(i\omega) \cdot G(i\omega) \cdot H(i\omega)\right| = 1$
- Phase margin: phase when $\left|F(i\omega) \cdot G(i\omega) \cdot H(i\omega)\right| = 1$





Example: 2D airfoil



$$\begin{bmatrix} 1 & x_\alpha \\ x_\alpha & r_\alpha^2 \end{bmatrix} \begin{Bmatrix} \dfrac{\ddot{h}}{c/2} \\ \ddot{\alpha} \end{Bmatrix} + \omega_\alpha^2 \begin{bmatrix} \left(\dfrac{\omega_h}{\omega_\alpha}\right)^2 & 0 \\ 0 & r_\alpha^2 \end{bmatrix} \begin{Bmatrix} \dfrac{h}{c/2} \\ \alpha \end{Bmatrix} =$$

$$= \frac{q_\infty}{M} \begin{bmatrix} 0 & -4\pi \\ 0 & 4\pi\left(\dfrac{1}{2}+\Lambda_1\right) \end{bmatrix} \begin{Bmatrix} \dfrac{h}{c/2} \\ \alpha \end{Bmatrix} +$$

$$+ t_0 \frac{q_\infty}{M} \begin{bmatrix} -4\pi & -4\pi\left(\dfrac{1}{2}-\Lambda_1\right) \\ 4\pi\left(\dfrac{1}{2}+\Lambda_1\right) & 4\pi\left(\dfrac{1}{4}-\Lambda_1^2\right) \end{bmatrix} \begin{Bmatrix} \dfrac{\dot{h}}{c/2} \\ \dot{\alpha} \end{Bmatrix} +$$

$$+ \frac{q_\infty}{M} \begin{Bmatrix} -4T_{10} \\ 4T_{10}\left(\dfrac{1}{2}+\Lambda_1\right) \end{Bmatrix} \delta_c - \begin{Bmatrix} x_\delta \\ r_\delta^2 + \Lambda_{31} x_\delta \end{Bmatrix} \ddot{\delta}_c$$

Proportional-derivative feedback system

$$\left[\hat{M}_{ij}\right]\{\ddot{u}_h\} + \omega_\alpha^2\left[\hat{K}_{ij}\right]\{u_h\} \approx \frac{1}{2\pi\mu t_0^2}\left[\hat{Q}_{ij}^0\right]\{u_h\} + \frac{1}{2\pi\mu t_0}\left[\hat{Q}_{ij}^1\right]\{\dot{u}_h\} + \\ + \frac{q_\infty}{2\pi\mu t_0^2}\left\{\hat{Q}_{ic}\right\}\delta_c.$$

↓

$$\delta_c = K_p \left\lfloor\, a_1 \quad a_2 \,\right\rfloor \left\{\begin{array}{c} h \\ \hline c/2 \\ \alpha \end{array}\right\} + t_0 K_d \left\lfloor\, b_1 \quad b_2 \,\right\rfloor \left\{\begin{array}{c} \dot{h} \\ \hline c/2 \\ \dot{\alpha} \end{array}\right\} = \\ = K_p \left\lfloor\, a_1 \quad a_2 \,\right\rfloor \{u_h\} + t_0 K_d \left\lfloor\, b_1 \quad b_2 \,\right\rfloor \{\dot{u}_h\},$$

↓

$$\left[\hat{M}_{ij}\right]\{\ddot{u}_h\} + \omega_\alpha^2\left[\hat{K}_{ij}\right]\{u_h\} = \frac{1}{2\pi\mu t_0^2}\left[\hat{Q}_{ij}^0\right]\{u_h\} + \frac{1}{2\pi\mu t_0}\left[\hat{Q}_{ij}^1\right]\{\dot{u}_h\} + \\ + \frac{1}{2\pi\mu t_0^2}\left\{\hat{Q}_{ic}\right\}\left(K_p \left\lfloor\, a_1 \quad a_2 \,\right\rfloor \left\{\begin{array}{c} h \\ \hline c/2 \\ \alpha \end{array}\right\} + t_0 K_d \left\lfloor\, b_1 \quad b_2 \,\right\rfloor \left\{\begin{array}{c} \dot{h} \\ \hline c/2 \\ \dot{\alpha} \end{array}\right\}\right) = \\ = \frac{1}{2\pi\mu t_0^2}\left[\hat{Q}_{ij}^0\right]\{u_h\} + \frac{1}{2\pi\mu t_0}\left[\hat{Q}_{ij}^1\right]\{\dot{u}_h\} + \frac{1}{2\pi\mu t_0^2}[G_p]\{u_h\} + \frac{1}{2\pi\mu t_0}[G_d]\{\dot{u}_h\},$$

↓

$$\left[\hat{M}_{ij}\right]\{\ddot{u}_h\} - \frac{1}{2\pi\mu t_0}\left(\left[\hat{Q}_{ij}^1\right] + [G_d]\right)\{\dot{u}_h\} + \\ + \left[\omega_\alpha^2\left[\hat{K}_{ij}\right] - \frac{1}{2\pi\mu t_0^2}\left(\left[\hat{Q}_{ij}^0\right] + [G_p]\right)\right]\{u_h\} = 0$$

$$\left\{\begin{array}{c} L_g \\ M_{ACg} \end{array}\right\} = q_\infty c \left\{\begin{array}{c} C_{L\alpha} \\ eC_{L\alpha} \end{array}\right\} \frac{w_g}{U_\infty}$$

$$\begin{aligned}
\left\{\begin{array}{c} \dfrac{L_g}{M\left(c/2\right)} \\ \dfrac{M_{ACg}}{M\left(c/2\right)^2} \end{array}\right\} &= \frac{\frac{1}{2}\rho_\infty U_\infty^2}{M} \left\{\begin{array}{c} 2C_{L\alpha} \\ 2\frac{e}{c}C_{L\alpha} \end{array}\right\} \frac{w_g}{U_\infty} = \\
&= \frac{1}{2\pi \dfrac{M}{\pi\rho_\infty \left(\frac{c}{2}\right)^2} \left(\dfrac{c}{2U_\infty}\right)^2} \left\{\begin{array}{c} 2C_{L\alpha} \\ 2\frac{e}{c}C_{L\alpha} \end{array}\right\} \frac{w_g}{U_\infty} = \\
&= \frac{1}{2\pi\mu t_0^2} \left\{\hat{Q}_{ig}\right\} \hat{w}_g .
\end{aligned}$$

$$\begin{aligned}
&\left[\hat{M}_{ij}\right] \left\{\ddot{u}_h\right\} - \frac{1}{2\pi\mu t_0} \left(\left[\hat{Q}_{ij}^1\right] + \left[G_d\right]\right) \left\{\dot{u}_h\right\} + \\
&+ \left[\omega_\alpha^2 \left[\hat{K}_{ij}\right] - \frac{1}{2\pi\mu t_0^2} \left(\left[\hat{Q}_{ij}^0\right] + \left[G_p\right]\right)\right] \left\{u_h\right\} \quad = \quad \frac{1}{2\pi\mu t_0^2} \left\{\hat{Q}_{ig}\right\} \hat{w}_g .
\end{aligned}$$

$$\begin{aligned}
\frac{\left\{\tilde{u}_h\right\}}{\hat{w}_0} &= H_{ig}\left(\omega\right) = \\
&= \frac{1}{2\pi\mu t_0^2} \left[-\omega^2 \left[\hat{M}_{ij}\right] + \omega_\alpha^2 \left[\hat{K}_{ij}\right] - \right. \\
&\left. -\mathrm{i}\omega \frac{1}{2\pi\mu t_0^2} \left(\left[\hat{Q}_{ij}^0\right] + \mathrm{i}\omega t_0 \left[\hat{Q}_{ij}^1\right] + \left[G_p\right] + \mathrm{i}\omega t_0 \left[G_d\right]\right) \left\{\hat{Q}_{ig}\right\}\right.
\end{aligned}$$

$$PSD_{OUT} = \left|H_{ig}\left(\omega\right)\right|^2 PSD_{CT}$$

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